Inverted Pendulum on a Cart

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Abstract

We decided to investigate the movement of an inverted pendulum on a cart. In addition, we wanted to control the inverted pendulum and stabilize it.

Concept Outline:

In order to create a control for our inverted pendulum, a model of the given system had to be created. Once we created the equations of motion, we tested how our model behaves, and input an external force to act on the cart. Afterwards, we created a simple control to keep our pendulum upright.

I. Our Model

Our system involves two changing variables--X and θ. X, the location of the cart relative to the origin and θ, the angle of the pendulum in relationship to the cart will change depending on different variables. The constant variables are length(L) and mass (m & M). In our particular model, we chose a length of 2 meters, and 1kg for each of the masses.

We also implemented an outside force on the cart in our model. This force is used later on to control the motion of the pendulum.

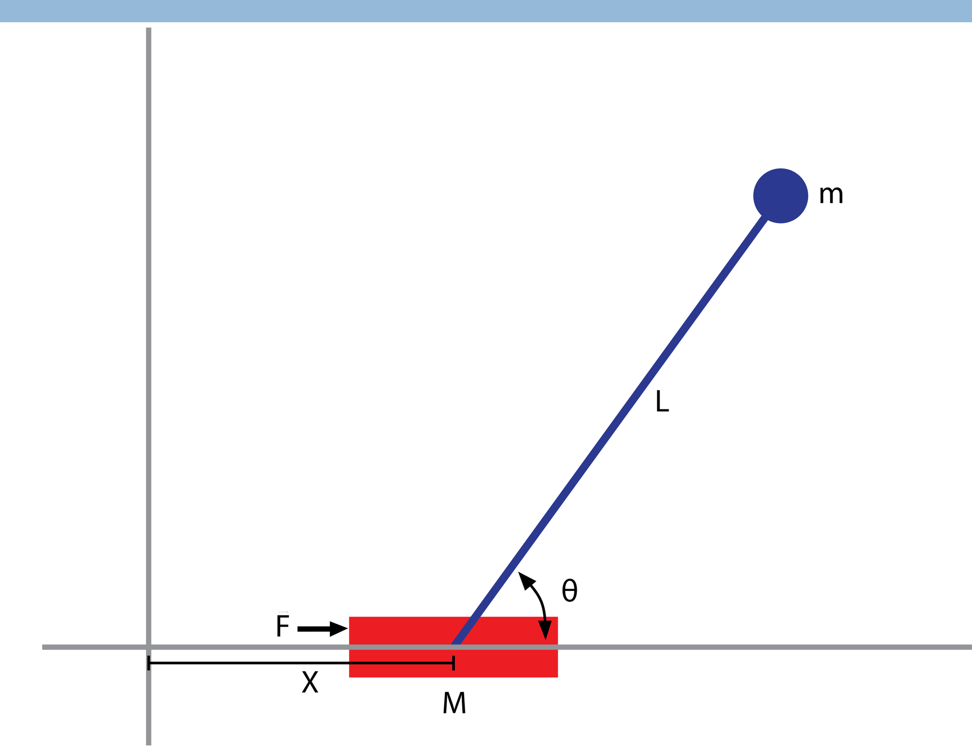


Diagram of the System

II. Workings of the Simulation

Using Lagragian mechanics and Newton's laws of motions, we created equations relating the angle of the pendulum to the movement of the cart. The bolded terms (force) are used when modeling the control of the pendulum.

$$\ddot{x} = \frac{m\ell\dot{\theta}^2 cos\theta - mgsin\theta cos\theta + \boldsymbol{F}}{M+m-msin^2\theta} \qquad \ddot{\theta} = \frac{m\ell\dot{\theta}^2 cos\theta sin\theta - Mgcos\theta - mgcos\theta + \boldsymbol{F sin\theta}}{M\ell+m\ell-m\ell sin^2\theta}$$

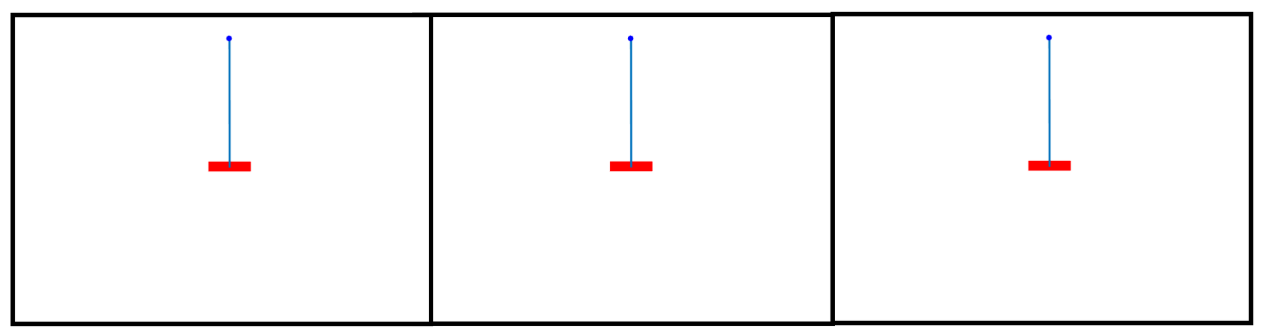
III. Limitations of Model

Our model excludes all types of friction: friction between the pivot point of the cart and the pendulum, friction between the cart and the floor, as well as air resistance. Friction would affect how our model acts, as an inverted pendulum is a negative feedback loop. Any perturbation will be magnified.

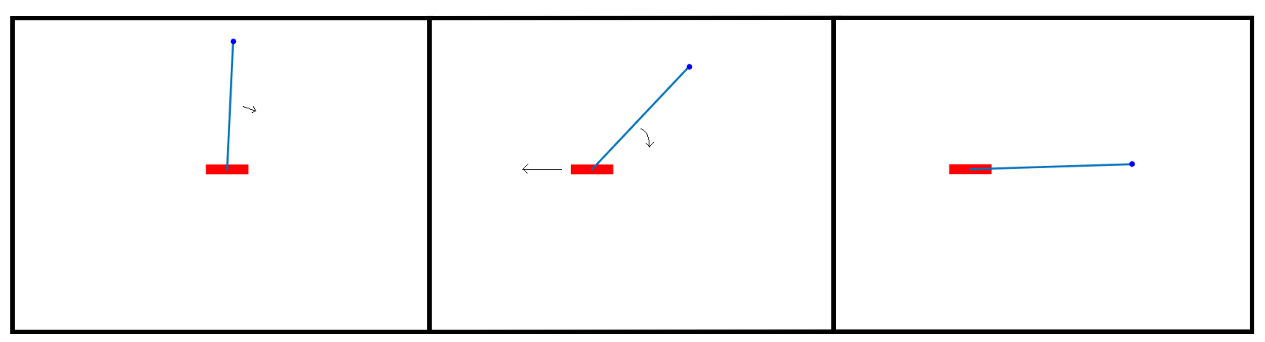
Further, our model does not account for inertia from the wheels, as well as rotational inertia of the rod. Inertia would make it more difficult for the cart the change direction and react to the movements of the pendulum.

IV. Model Prediction

Because we modeled an ideal world, no energy is lost and the system should move in a consistent pattern. After simulating the motion, we saw that the system does behave as predicted, swinging back and forth across a single point. It accelerates following laws of gravity and maintains a fixed distance between cart and pendulum. In order to test our model further, we varied our initial conditions to observe how our model acts.

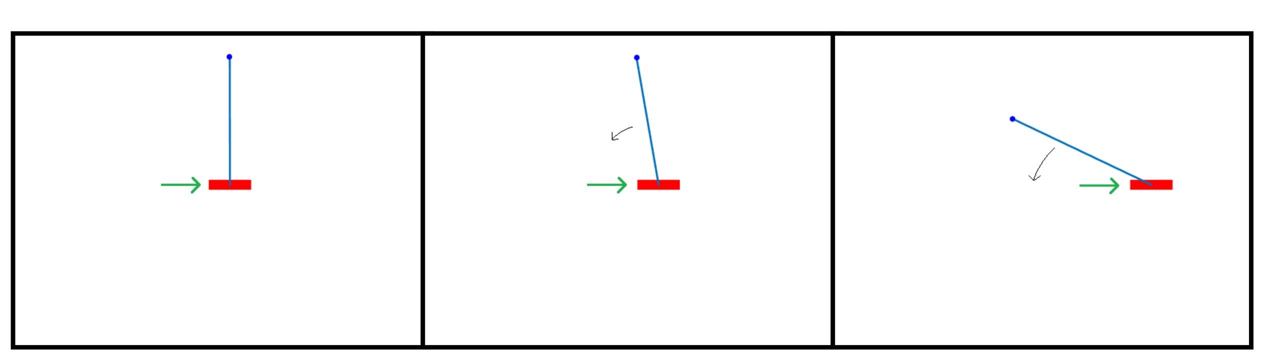


Motion with no offset. The pendulum stays in its upright position.

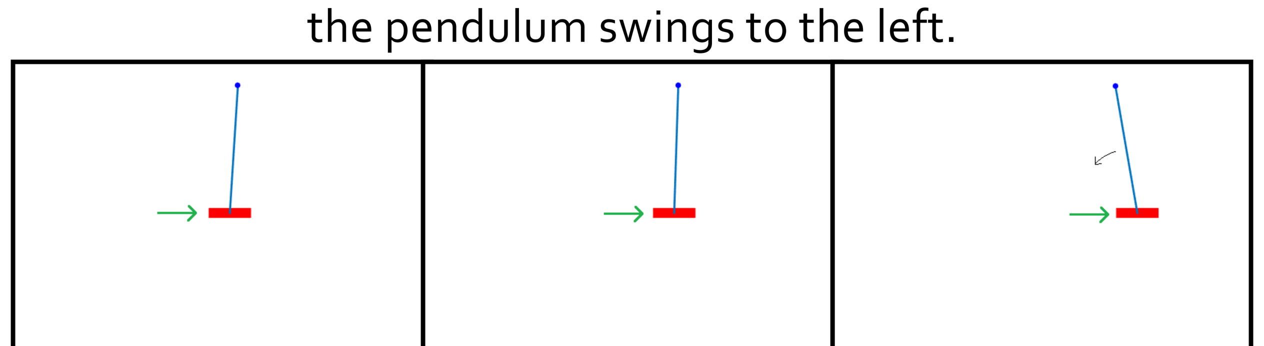


Motion with offset of 1 degree. The pendulum falls to the right and moves leftward accordingly due to the offset.

We further validated our model by applying constant external forces to the cart and seeing how our model behaves. Since our equations were created to include this force, the energy should still be conserved within the condition. The addition of this force should allow us to verify the model to ensure that in all conditions, the model should behave as expected.



Motion with force of 1N. As the cart rolls to the right, and the pendulum swings to the left.



Motion with force of 1N in addition to an offset. The pendulum maintains its position, but in the end falls over.

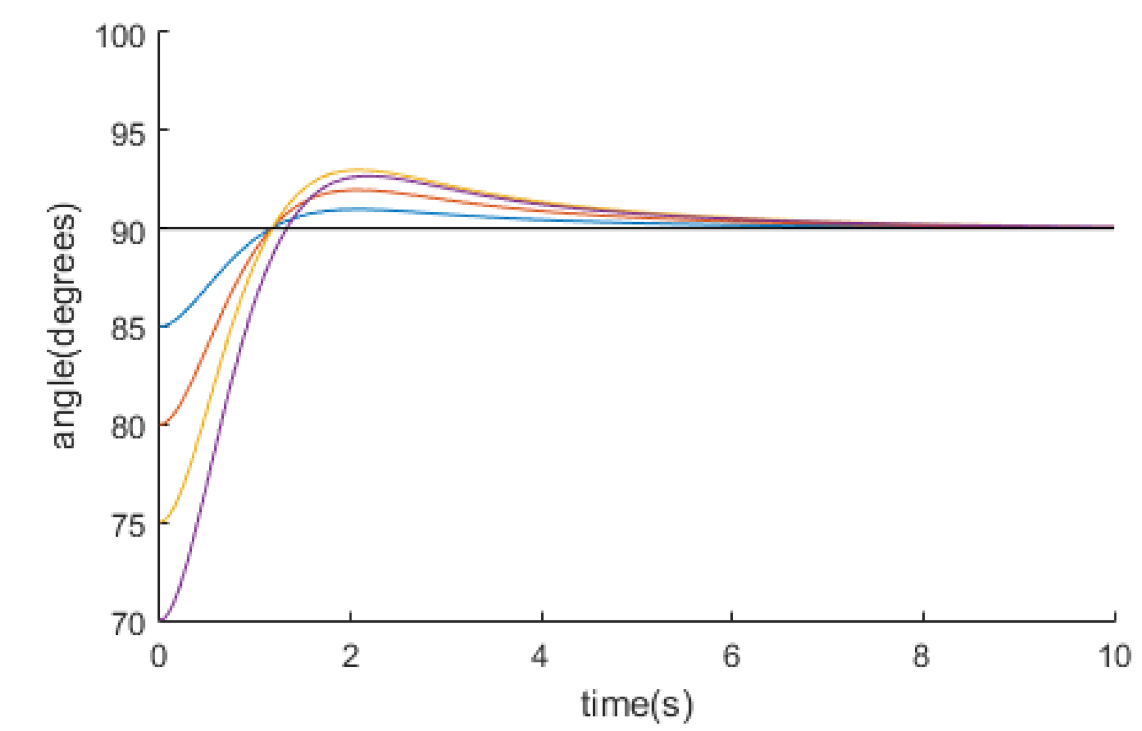
V. Control

Using the angle of the pendulum relative to the cart, we were able to correspondingly adjust the force applied on the cart in order to achieve the desired results.

We used a PID control loop, which controls the cart's movement based on different quantities. A PID loop tries to minimize the error of the pendulum, and minimize the overshoot of the pendulum by tuning each of the three constants.

$$F = K_p\theta + K_d\dot{\theta} + K_i\int\theta$$

VI. Success!

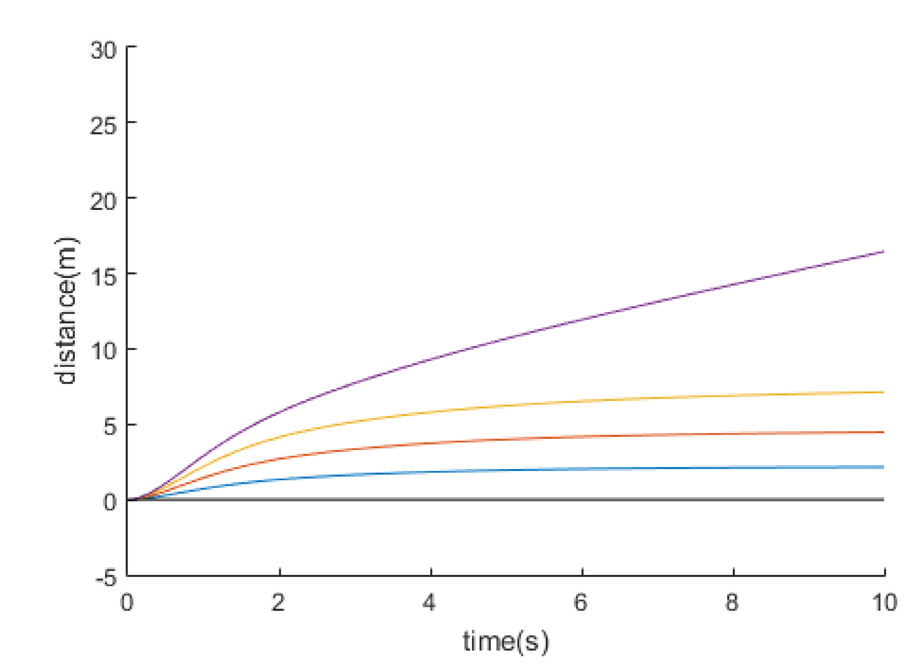


Angle of Pendulum returns to 90 degrees

For varying initial conditions (70-85 degrees), the force on the cart is automatically adjusted to return the pendulum to an upright position.

VII. Limitations of Control

The limitations of this control mechanism is that this PID control loop ignores the position of the cart. We cannot move the cart to a pre-determined location. We plotted the same initials for time vs. displacement:



Even though the pendulum returns to a value of 90 degrees, the cart continues to move.